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THE APPLICABILITY OF A MODEL OF SHEAR DELAMINATIONS†

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For the purpose of ascertaining the limits of applicability of a previously proposed model [1] of non-propagating shear delaminations for analysing a three-layer rod with delamination-type defects, the numerical solution of the corresponding three-dimensional problem in the theory of elasticity is constructed and analysed using Version 6.1 of the ANSYS (Analysis of Systems) finite-element software package. The limits of applicability of the proposed delamination model are estimated.

An analysis of the present state of research on the problem of strength of thin-walled multilayer structures with defects of the delamination type can be found in [2]. A new model of non-propagating shear delaminations for analysing a three-layer rod with delaminations was proposed in [1] and extended to the case of three-layer plates with delaminations in [3]. The model developed in [1, 3] extends the well-known Kirchhoff model, Timoshenko's shear model and Grigolyuk's broken line model [4] to three-layer structures with shear-type delaminations and transverse compression of the filler [5].

An estimate of the reliability of the hypotheses concerning the deformation of a three-layer rod with shear-type delaminations, used when constructing the model [1], is made by analysing the results of numerical calculations.

1. THE MODEL OF SHEAR DELAMINATION

We will first discuss the fundamental hypotheses of the shear delamination model which has been adopted. The deflection accompanying the thermal force and the static loading of a three-layer rod of asymmetric structure throughout its thickness and with delamination-type defects on the surfaces of contact of the layers is considered. The outer layers are assumed to be thin, elastic and isotropic, of different thickness, and the hypothesis of planar sections of a Bernoulli rod is assumed to hold for them. The intermediate central layer, that is, the filler, is assumed to be a transversely isotropic layer which is sensitive to the longitudinal and transverse forces and moments and which also transfers transverse shear and is compressed in the transverse direction.

The three-layer rod is acted upon by transverse loads $q_1(x)$ and $q_2(x)$, which are applied to the load-bearing layers, and is non-uniformly heated along its length and thickness up to a specified temperature $T(x, z)$ (Fig. 1).

The following notation is introduced: x is the longitudinal coordinate $(0 < x < l)$, z is the transverse coordinate, h_1, h_2 and $h_3 = 2c$ are the thicknesses of the first and second load-bearing layers and the filler respectively, and $\Delta x^k = x_2^k - x_1^k$ is the length of the delamination ($k = 1, 2$ are the numbers of the delaminations).

Broken-line hypotheses are insufficient to describe the deformation of a three-layer rod with a compressible filler and shear-type delaminations, and additional assumptions have to be introduced.

In order to take account of the compressibility of the filter, a linear distribution of the normal displacements over the thickness of the middle layer [5]

$$w^3 = w + \frac{z}{c}\upsilon; \quad w = \frac{w^1 + w^2}{2}; \quad \upsilon = \frac{w^1 - w^2}{2}$$

is adopted, where w^k is the deflection of the kth layer ($k = 1, 2, 3$) and υ is a function which characterizes the compression of the filler. In the case of an incompressible filler, $w^1 = w^2$ and $\upsilon = 0$.

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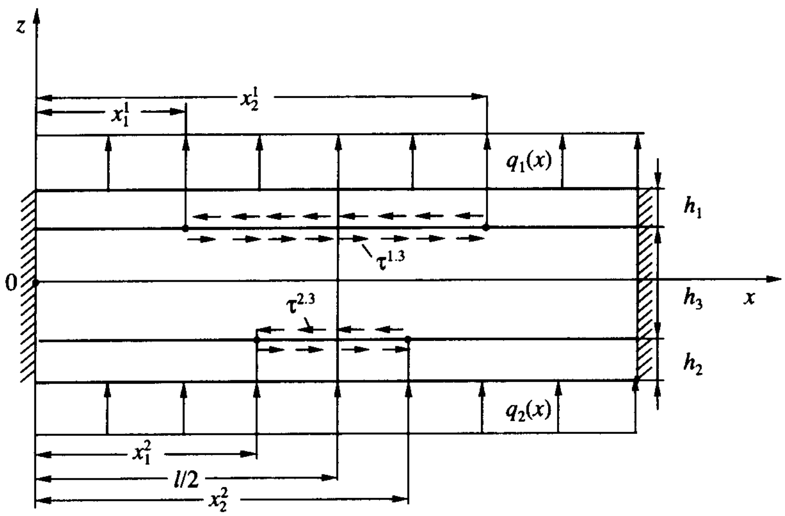

Fig. 1

It is assumed that there are defect zones in the surfaces of contact of the outer layers with the filler which are simulated by non-propagating shear delaminations. These are characterized solely by a discontinuity in the longitudinal components of the displacement vector.

At the boundaries of contact of the load-bearing layers with the filler, the normal displacements are assumed to be continuous in the whole of the domain and the tangential displacements are continuous so long as the transverse shear stresses σ_{xz}^{k3} ($k = 1, 2$), which are transferred in the delamination zones from the load-bearing layers by the filler, do not exceed the limiting permissible values for the filler material, which are given by the Coulomb–Amonton law

$$\sigma_{xz}^{k3} \leq [\sigma_{xz}^{k3}] + f_k \sigma_z^3$$

where $[\sigma_{xz}^{k3}]$ are the specified limiting values of the shear stresses for a given material, which characterize the cohesion of the contacting surfaces which is independent of the normal pressure, f_k are the friction coefficients and σ_z^3 are the normal stresses at the boundaries of contact of the filler with the load-bearing layers.

After the interlayer shear stresses σ_{xz}^{k3} exceed the permissible value, slippage occurs between the layers (mutual displacements of the layers in the delamination zones) and interlayer shear forces τ^{k3} ($k = 1, 2$) appear (Fig. 1).

By analogy with the theory of composite rods with elastically-yielding shear couplings, it is assumed that, in view of the smallness of the deformations, the interlayer shear forces are proportional to the mutual displacements of the layers in the delamination zones.

$$\tau^{k3}(x) = \lambda_k(u^k - u^3)\Big|_{z = (-1)^{k+1}c} \quad x \in [\Delta x^k], \quad k = 1, 2$$

where λ_k are specified constants which have a similar meaning to the coefficients of rigidity of seams in the theory of composite rods, and u^k are the longitudinal displacements of the kth load-bearing layer.

These relations can be extended to the case where account is taken of the non-linear coupling between the forces τ^{k3} and the mutual displacements of the layers in the delamination domains. Only linear relations are analysed in this paper.

The longitudinal displacements of the layers of a three-layer rod for the defect-free and defective domains are given by the following formulae

$$u^1 = u_1 - (z - c)\frac{dw^1}{dx}, \quad c \leq z \leq c + h_1; \; u^2 = u_2 - (z + c)\frac{dw^2}{dx}, \quad -c - h_2 \leq z \leq -c$$

$$u^3 = u + z\left(\alpha^3 - \frac{dw}{dx}\right) - \frac{z^2}{2c}\frac{dv}{dx}, \quad -c_2 \leq z \leq c$$

where u_1, u_2 and u are the longitudinal displacements of the middle line of the load-bearing layers and the filler and $\alpha^3 = \varepsilon^3_{xz}$ is the angle of transverse shear in the filler.

By virtue of the continuity of the longitudinal displacements in the defect-free domain, we have

$$u^1 = u^3\big|_{z=c} = u + c\left(\alpha^3 - \frac{dw}{dx}\right) - \frac{c}{2}\frac{d\upsilon}{dx}, \quad u^2 = u^3\big|_{z=-c} = u - c\left(\alpha^3 - \frac{dw}{dx}\right) - \frac{c}{2}\frac{d\upsilon}{dx}$$

Hence, in accordance with the hypotheses which have been adopted, the longitudinal displacements u^i ($i = 1, 2, 3$) and the normal displacements w^i ($i = 1, 2, 3$) have the following distribution over the thickness of the three-layer structure.

On passing from layer to layer, the longitudinal displacements u^1, u^2, u^3 are continuous in the defect-free region and undergo a finite discontinuity in the shear delamination zones. The variation of the displacements u^1 and u^2 (the outer layers) over the thickness is linear and the variation of the displacements u^3 (the filler) is non-linear.

The normal displacements w^1, w^3, w^2 in the defect-free domains and in the zone of shear delamination are continuous on passing from layer to layer, constant throughout the thickness in the load-bearing layers and distributed linearly over the thickness of the filler.

It follows from the variational equation for the problem that the generalized displacements u^1, u^2, u, α^3, dw/dx, $d\upsilon/dx$, w, υ must correspond to the complete and generalized forces N^1, N^2, N, H, M, L, dM/dx, dL/dx of a three-layer rod with delaminations. The number of basic, independent unknowns in the problem of the deflection of a three-layer, axisymmetric loaded rod is therefore equal to 16.

Moreover, the unknown interlayer shear forces τ^{k3} of interaction of the layers in the delamination zones are determined in terms of the longitudinal displacements on the boundaries of contact of the defective domains.

Here, we shall confine ourselves to discussing a model of non-propagating shear delamination. The differential equations of the problem, the boundary conditions and the conditions for the solutions in the defective and defect-free zones to match have been presented and discussed in detail earlier [2, 3], and a system of governing equations in the normal Cauchy form for integration by the orthogonal sweep method has also been presented.

2. TESTING OF THE PROGRAM

The program was tested on the classical problem of the deflection, taking account of shear, of a prismatic rod of length l and rectangular cross-section ($b \times h$) acted upon by a uniform, transverse, external pressure q.

The theory of Bernoulli prismatic rods gives the following formulae for the magnitudes of the maximum deflection at the centre of a rod with two supports

$$w = w^b + w^s; \quad w^b = \frac{k}{384}\frac{Ql^3}{EJ}, \quad w^s = \frac{Ql}{8G\omega_c} \tag{2.1}$$

Here, $Q = qlb$ is the magnitude of the distributed external load, $J = bh^3/12$ is the moment of inertia of the cross-section area with respect to the neutral axis, $\omega_c = bh$ is the reduced cross-section area of the rod, E is Young's modulus, $G = E/(2(1 + \nu))$ is the shear modulus, and ν is Poisson's ratio; for a hinged rod $k = 5$ and for a rod rigidly clamped onto two supports, $k = 1$.

The one-dimensional solution for a Bernoulli rod is compared with the numerical solution of the three-dimensional problem in the linear theory of elasticity for a three-dimensional rod. A finite-element solution of the problem of the deflection of a three-dimensional, homogeneous, prismatic rod ($b \times h \times l$) of rectangular cross-section acted upon by a uniform pressure q was constructed. Version 6.1 of the ANSYS software package and the same technique for subdividing the volumes with hexagonal finite elements as in the following problem of the deflection of a rod with delamination were used.

The following dimensionless geometrical and stiffness characteristics were taken for the three-dimensional rod: the dimensionless length of the rod $L = 0.2 - 1.0$, the ratio of the sides of the rectangular cross-section of the rod was always $h/b = 4/5$, the ratio h/l was varied within the limits from 1/25 to 1/5, $E = 0.7 \times 10^7$ MPa and $\nu = 0.3$.

In the case of a relatively shorter rod from this series, in accordance with formulae (2.1), the contribution of the shear component is 8.3% in the case of a hinged support and 41.6% in the case of rigid clamping. As the length of the rod increases, the contribution of the shear component decreases as shown in Fig. 2, where the relative length of the rod L is plotted along the abscissa and the quantities

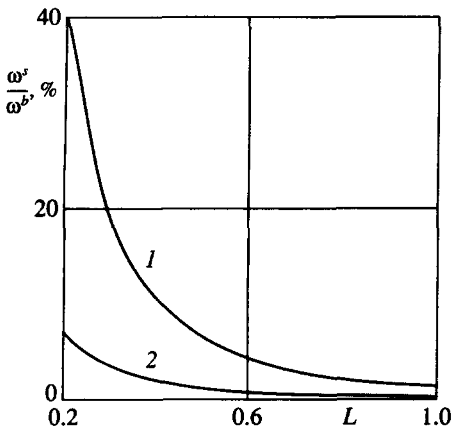

Fig. 2

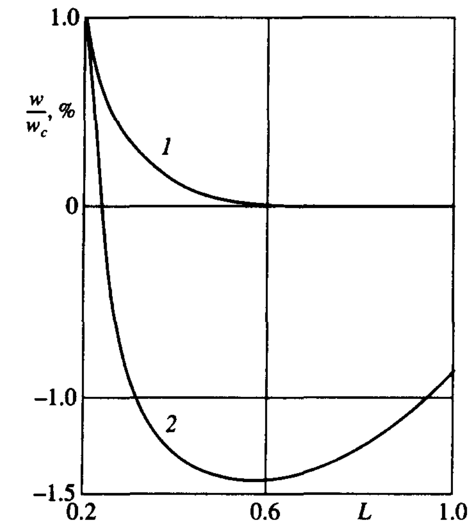

Fig. 3

w^s/w^b are plotted as percentages along the ordinate. Curve 1 corresponds to the case of the rigid clamping of the rod onto the supports and curve 2 corresponds to a hinged support.

The accuracy of the calculations of the deflections by the finite element method is higher for a hinged support than for a rigidly clamped rod. This is explained by the fact that the hinged support conditions are the natural boundary conditions of the corresponding variational problem. For beams with a length from $L = 0.2$ to $L = 1$, the difference between the rod solution for normal deflection and the three-dimensional, finite-element solution is fraction of a percent in the case of a hinged support and does not exceed 1.3% in the case of rigid clamping. A comparison of the results of the calculation of the maximum deflections using beam theory and the finite element method is given in Fig. 3, where the relative length of the rod is plotted along the abscissa and the ratio of the deflection at the centre of the span of the rod (w), according to formula (2.1), to the deflection (w_c), calculated by the finite element method, is plotted as a percentage along the ordinate axis. Curve 1 corresponds to the case of a hinged support and curve 2 to rigid clamping on supports.

Hence, the test calculations have shown that the finite element method used and the finite-element representation adopted adequately describe the deflection with shear of a homogeneous, three-dimensional rod without delaminations.

3. SOLUTION OF THE THREE-DIMENSIONAL PROBLEM OF THE LINEAR THEORY OF ELASTICITY FOR A ROD WITH DELAMINATION

The finite element method in the ANSYS 6.1 software package was used.

It is well known that the key factors in solving a problem using the finite element method are the choice of the type of finite element, the method of subdividing the volume into these elements and the method used to solve the systems of algebraic equations. In the case considered here, the problem is further complicated by the fact that a non-linear contact problem of the theory of elasticity has to be solved in the delamination zone. This problem is quite difficult even for such a powerful computational instrument.

Finite elements. SOLID45, 64 and 95 were adopted as the finite elements. The finite element SOLID45 is a three-dimensional, eight mesh point, hexagonal element which can be transformed into a prism and a tetrahedron. The mesh points are located at the hexahedron vertices and each of them has three degrees of freedom. This element allows one to study plasticity, creep, buckling, limiting stresses, large deflections and deformations for isotropic and orthotropic materials. The element SOLID64 is the analogue of SOLID45 for an anisotropic material.

Control calculations were carried out using the finite element SOLID95 which is a higher-order version of the element SOLID45. This is a three-dimensional, 20 mesh point, hexagonal element with 60 degrees of freedom for investigating problems in the mechanics of a deformable solid body.

TARGE170 and CONTA174 were taken as the contact finite elements.

Contact interaction parameters. The contact elements contain fourteen real constants by means of which the mechanical and geometrical properties of the contacting surfaces are described. Of these, two of the real constants have a greater effect on the convergence and the solution of the contact problem: f_{kn} is the stiffness of the contact along the normal and $f_{k\tau}$ is the tangential (shear) contact rigidity.

Using a special algorithm, the remaining 12 constants are either set default or calculated using the specified values of the constants f_{kn} and $f_{k\tau}$. The developers of the software also proposed a method of selecting the constants f_{kn} and $f_{k\tau}$ from a given range of values under the control of the convergence of the process and the results obtained. This approach to the selection of the constants is well known and fairly universal but requires constant monitoring of the results obtained.

Now, concerning the real constants of the shear model of delamination which is being studied [1]. In the contact module of the program, all three parameters of the adopted model of shear delamination are used: $[\sigma_{xz}^{k3}]$, the coefficient of cohesion, f_k, the friction coefficient and $\lambda_k = f_{k\tau}$, the shear contact rigidity. The constant f_{kn} does not appear among the basic parameters of the model [1] of shear delamination. Nevertheless, it remains fundamental in the investigation of the deformation of the structure with shear delamination being studied. This is explained by the fact that the deflection of three-layer structures with a compressible filler is being studied. Calculations show that the filler is substantially deformed in this case.

Analysis of the results obtained. A special investigation was undertaken into the choice of the finite-element subdivision using the computation time and the convergence of the iterative process as a control. In the case of axisymmetric deformation, a version in which all the longitudinal lines of a half of the rod were subdivided into 20 parts and each of the layers was subdivided along its thickness and width into ten parts was found to be the best. For this case, the number of mesh points was 7831 and the computation time was 16 minutes for a Pentium-type processor. The Newton–Raphson method was used to construct the iterative process for solving the non-linear contact problem and the method of conjugate gradients, with a relative computational error of 10^{-6}, was used to solve the system of algebraic equations.

The applicability of the shear delamination model proposed earlier in [1] was investigated for a relatively thick, three-layer rod when the effect of shear deformations are more pronounced. The moduli of elasticity of the load-bearing layers and the filler differed by three orders of magnitude. The calculations were carried out for two forms of boundary conditions: a hinged support on a displacing contour and rigid clamping on a fixed contour. Note the difficulties in formulating the boundary conditions, especially for the first case of the boundary conditions. Here, in the case of a relatively softer filler, the effects of the independent behaviour of the layers manifest themselves as in the deflection of a beam on an elastic foundation. In the case of a rod which is not very wide, extrusion of the filler in the middle section of the span is also observed. The calculations showed that, in the case when one end of a three-layer structure is clamped onto a support, it is necessary to introduce a special structure and to set up the boundary conditions for it.

As a result of the calculations, numerous graphs of the distribution of the strains and stresses along the width and length of a three-layer rod with a defect of the delamination type were obtained. Graphs of the distributions of the strains and stresses over the thickness of the rod were constructed for four cross-sections along its length. Graphs of the distribution of the displacements u, υ and w along the span of the three-layer rod are also presented. The corresponding distributions of the stresses σ_x, σ_y and σ_{xy} were obtained. Moreover, the three-dimensional distribution of all these quantities is given for the same structure, and its deformed and undeformed states are shown. Distributions of the forces and displacements on the contacting surfaces are presented and the deformation of the same contact surfaces is also shown.†

Here, we shall only present the most characteristic graphs for a short rod $h/l = 1/5$ when the delamination zone between the filler and the load-bearing layers occupies half the central part of the span of the rod. We will first give the distribution of the normal displacements w. The lateral surface of half the length of a three-layer rod in the deformed and undeformed states (the dashed lines) is shown in Fig. 4 and the isolines of the normal displacements w, labelled with the numbers 1–8, are presented. The distribution of the deflections w in Fig. 4 enables us to estimate the applicability of the broken-line hypothesis when analysing three-layer rods with delaminations in the defective and the defect-free domains. Thus, in the case of the rod under consideration, this hypothesis is inapplicable in the defect-

†For details, see *Mamai, V. I.*, The possibilities of a numerical experiment for estimating the applicability of a model of shear delamination of thin-walled structures. Report No. 4593, Scientific Research Institute of Mechanics, Moscow State University, Moscow, 2001.

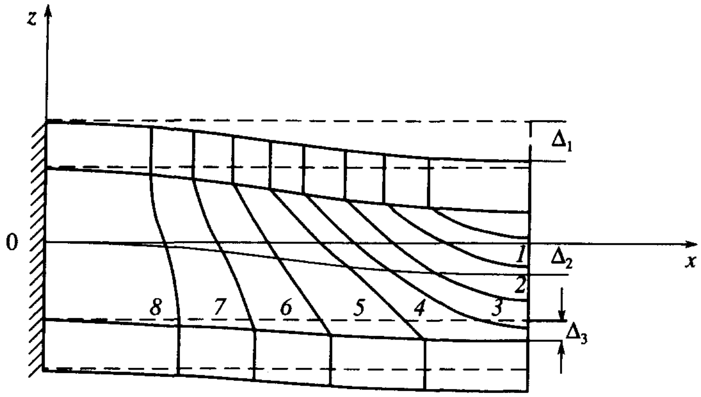

Fig. 4

free domain in the zone where the rod is fixed and in a large part of the shear delamination zone. We now turn our attention to the characteristic distribution of the deflections in the middle part of the span of the rod, which is explained by the compression of the filler. The relations obtained between the deflections of the layers of the rod in its central part (Fig. 4).

$$\Delta_1 \geqslant \Delta_2 \geqslant \Delta_3$$

are also explained by the compression of the filler.

We will denote the displacements u, υ, w at the point (x, y, z) normalized by the maximum value of the displacements w at the point $(x, 0, z)$ by $\tilde{u}$, $\tilde{\upsilon}$, $\tilde{w}$.

In the following two figures, we show the distributions of the displacements $\tilde{u}$, $\tilde{\upsilon}$, $\tilde{w}$ over the thickness of the package in the plane of symmetry of the rod (a) and on its lateral surface (b) in the defect-free domain when $x = 0.25\, L/2$ (Fig. 5) and in the domain of delamination when $x = 0.75\, L/2$ (Fig. 6). The dashed lines correspond to the surfaces of contact of the load-bearing layers of the filler. The dimensionless thickness of the rod H is plotted along the abscissa axis and the dimensionless displacements $\tilde{u}$, $\tilde{\upsilon}$, $\tilde{w}$ are plotted along the ordinate axis. It follows from these graphs that $\tilde{\upsilon}$, throughout the whole thickness of the package, and $\tilde{w}$, in the domain of the filler, undergo the greatest changes on passing from the plane $(x, 0, z)$ to the lateral surface of the rod. In the zone of shear delamination, $\tilde{u}$ and $\tilde{\upsilon}$ lose the discontinuity at the join of the filler and the load-bearing layers.

In conclusion, we present (Fig. 7) a typical graph of the distribution of the longitudinal displacements u over the thickness in the plane of symmetry of the rod for the section $x = 0.75\, L/2$ (in the delamination zone). The dimensionless height of the rod is plotted along the abscissa axis, and the ratio of the displacement υ along the x-axis to the value of the maximum displacement $\upsilon_{\max}$ at the join of the filler and the outer load-bearing layer is plotted along the ordinate axis. As before, the domains of contact of the outer layers and the filler are shown by dashed lines.

An analysis of the distribution of the displacements w and u over the thickness of the three-layer laminate which have been presented shows that the kinematic hypotheses adopted in [1] basically hold both in the domains with delaminations as well as in the defect-free domains. However, they are noticeably violated in the most deformed parts of the rod and, also, on its lateral surfaces.

4. MODAL ANALYSIS

The ANSYS 6.1 software package also enables one to study the modes of vibration of a prestressed structure. In the case of the deflection of a three-layer rod with shear-type delamination considered here, such a modal analysis enables one, to some extent, to make a judgement concerning the cooperative work of the layers in the shear delamination zone.

The problem of the stress–strain state of the three-layer rod with delaminations being studied, that is, the problem of deflection and the contact problem in the shear lamination zone, is initially solved. The first ten modes and frequencies of the natural oscillations of the structure are then calculated by the methods of modal analysis. The results of the calculations show that those modes of vibration of the filler in the shear delamination for which there is no opening of the contacting surfaces, correspond to the lower natural frequencies, which suggests that the hypothesis concerning the continuity of the normal displacement w in the shear delamination zone holds.

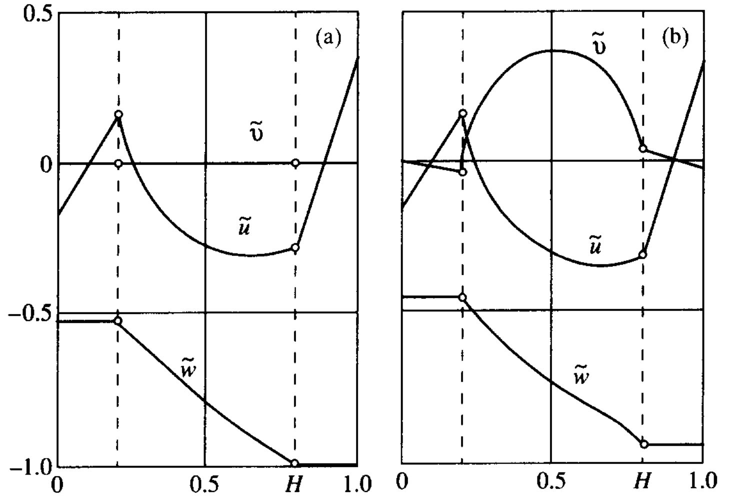

Fig. 5

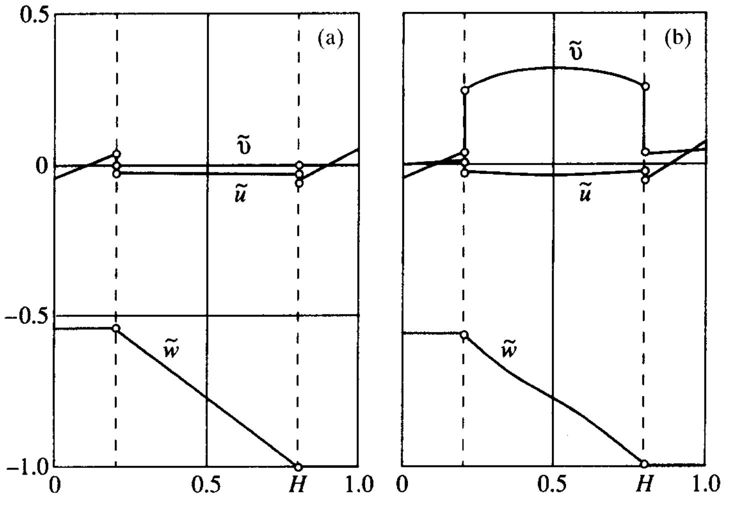

Fig. 6

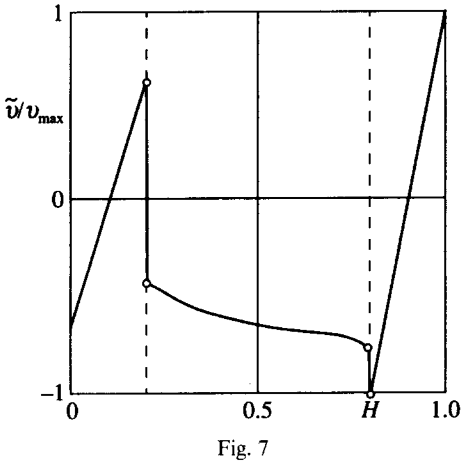

Fig. 7

5. CONCLUSIONS

1. The three-dimensional problem of the deflection of a three-layer three-dimensional rod with delamination has been solved by the finite element method using the ANSYS 6.1 software packages which, after the analysis which has been carried out, can be considered as a test problem in estimating the applicability of a previously proposed model of the shear delaminations of three-layer structures.

2. It has been established that, in the case of relatively shot rods ($h/l \geq 0.25$), the proposed model of a three-layer rod with delamination is inapplicable mainly because of the possibility of extrusion of the filler in the lateral surfaces of the three-layer rod. It is therefore undesirable to use this model when $h/l \geq 0.15$ for the class of load-bearing layer and filler materials considered.

3. It is better to use the finite element method when analysing shorter rods and it is desirable to analyse the rod together with the mechanical device for clamping the elements of the three-layer laminate to the support.

4. The broken-line hypothesis is satisfied quite well with the exception of the clamping region and in the domains where the phenomenon of extrusion of the filler is important.

5. In the shear delamination domain, the normal displacements do not suffer a discontinuity, but the tangential displacements suffer a discontinuity as the adopted model of delamination predicts.

6. The rigid, load-bearing layers of the three-layer, short rod behave in accordance with the scheme for Bernoulli rods.

7. The modal analysis carried out has shown that the hypothesis concerning the continuity of the normal displacements w holds in the shear delamination domain.

8. Shear delaminations over an area of 10–15% in the middle part of the span of the rod only have a slight effect on the magnitude of its normal deflections.

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